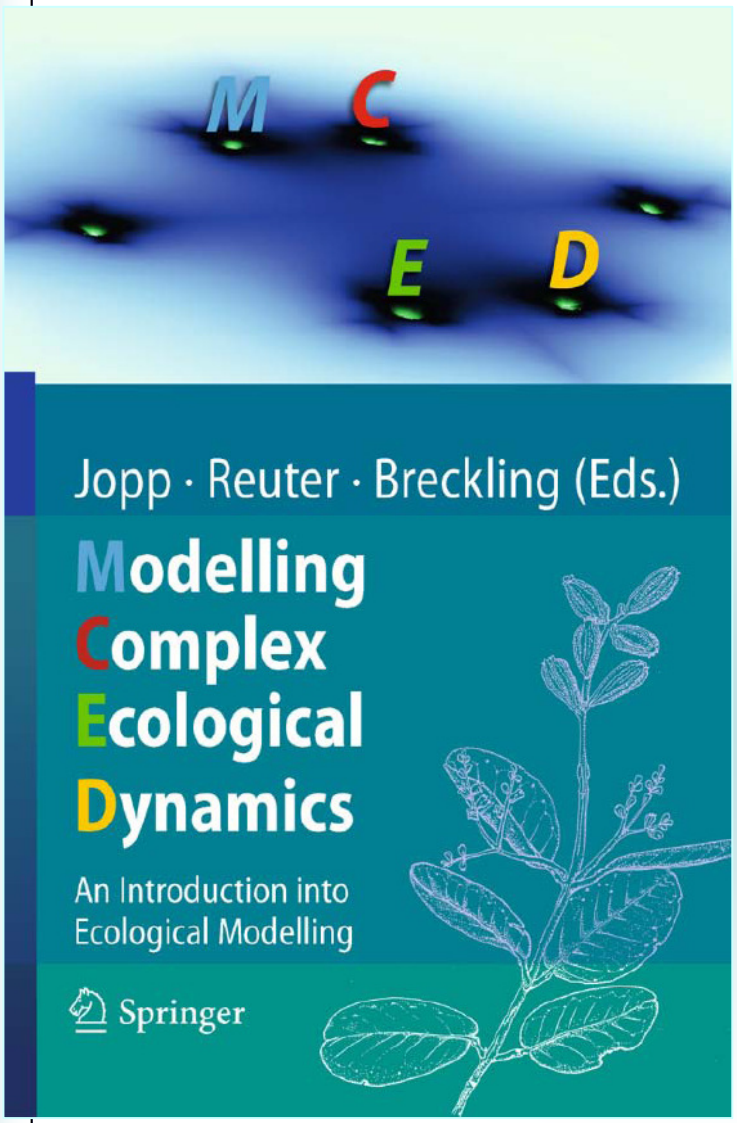
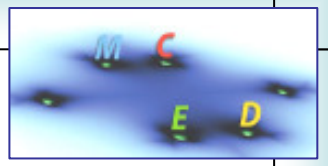
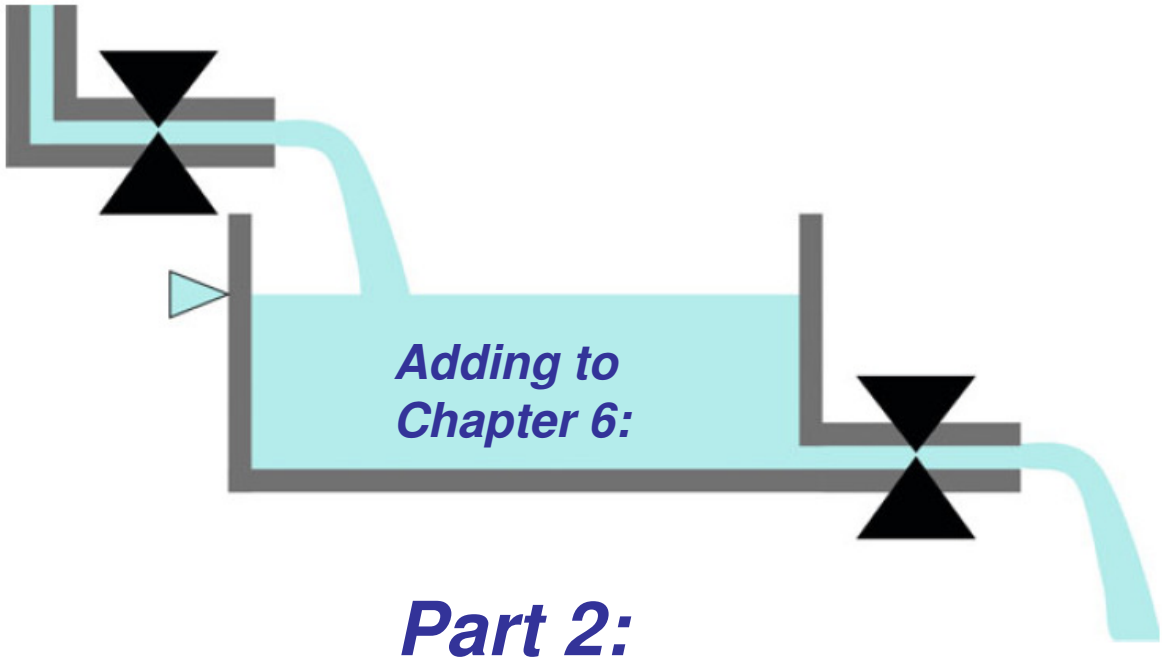


Presentation material accompanying

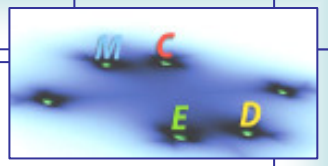
Chapter 6: Differential Equations



Modelling Complex Ecological Dynamics



Part 2: Difference Equations (Discrete systems)



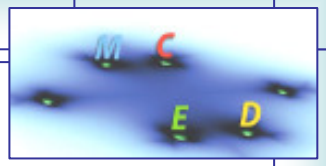
Preface

- *The presentation is free for use in non-commercial teaching context.*

- *The content was based on lecture material developed (among others) for the course "Systems Analysis" in the Master of Science Programme "International Studies in Aquatic Tropical Ecology" at the University of Bremen during the years 1999 – 2011*

- *Because of the page restriction this presentation partially extends the content covered in MCED Chapter 6 Differential Equations.*

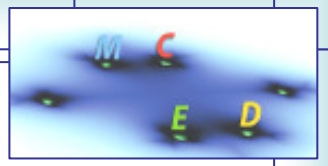
Broder Breckling
Hauke Reuter
Uta Berger



Difference Equations

(Discrete Systems)

plus Group Exercises



Difference equations (discrete systems)

previous step $\xrightarrow{calculation}$ succeeding step

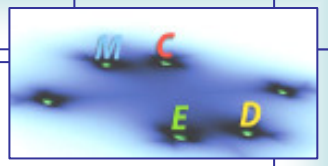
(no matter what is happening in between)

- arbitrary numbers of variables are possible
- example: dynamics of non-overlapping generations (e.g. butterflies)

initial state: x_o

current state: x_n

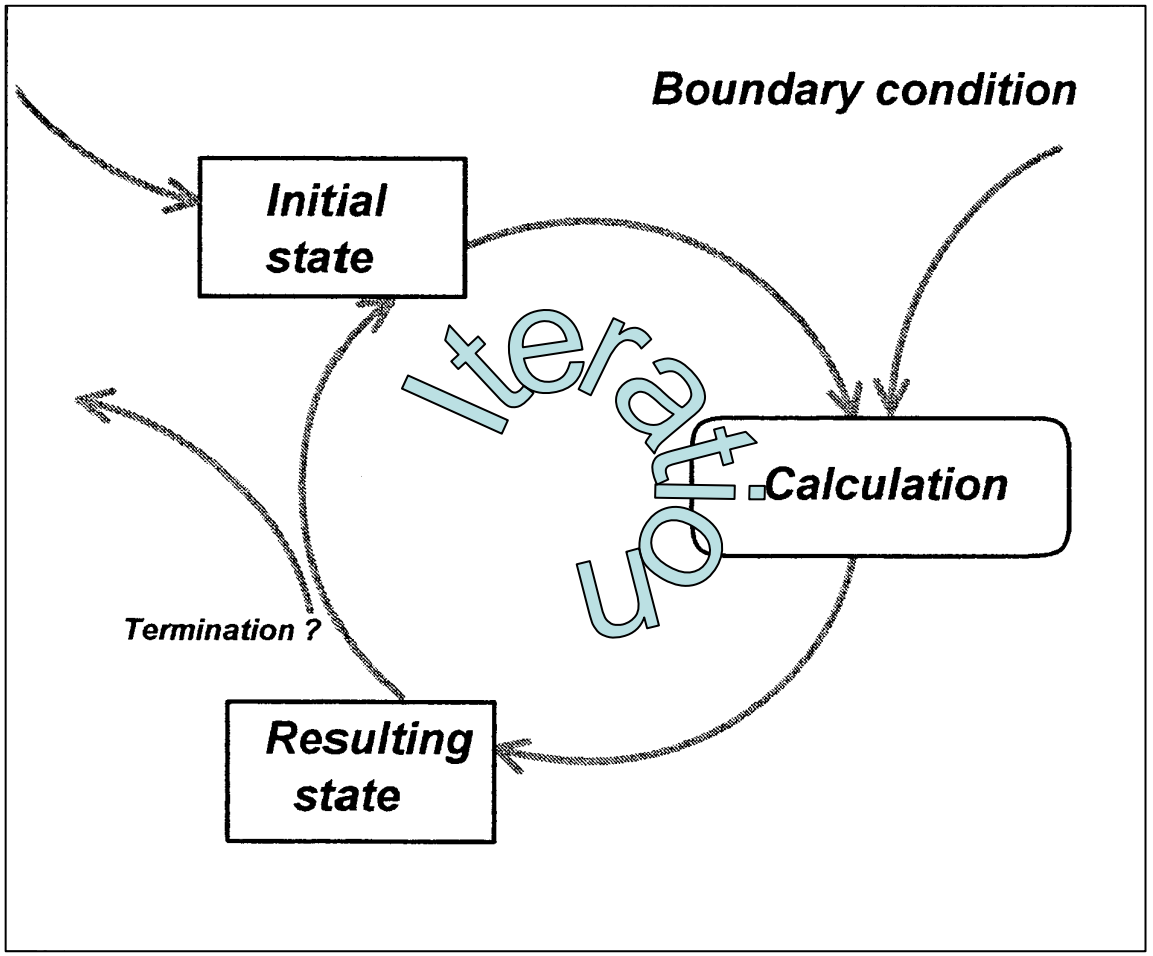
following state: x_{n+1} calculation: $x_{n+1} = f(x_n)$

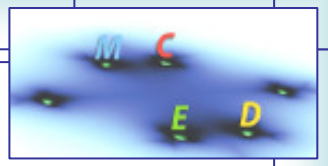


Difference equations (discrete systems)

Iteration:

proceeding from one state to the next, applying functional rules given through the relations between the elements





Difference equations (discrete systems)

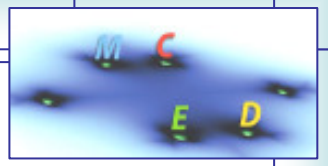
Example: Discrete exponential population growth

N size of the population

N_t size of population at time t

t = 0 indicates the starting point

initial population size: $N_0 = 500$



Difference equations (discrete systems)

Model description:

$$N_{t+1} = N_t + B - D + I - E$$

$$N_{t+1} - N_t = N_t - N_t + B - D + I - E$$

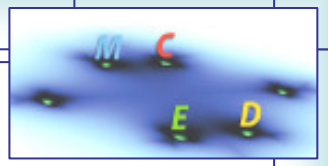
$$\Delta N = B - D + I - E$$

B = birth

D = death

I = immigration

E = emigration

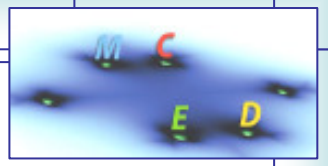


Difference equations (discrete systems)

Model assumptions:

1. **population is "closed" (no I or E; or I = E)**
2. **constant B and D**
3. **no genetic structure**
4. **no age and size structure**
5. **non-overlapping generations (discrete population growth)**

$$\Delta N = B - D + I - E \longrightarrow \Delta N = B - D$$



Difference equations (discrete systems)

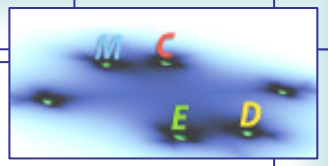
Another expression
(e.g., the population increases annually 36%, $r_d = 0.36$)

$$N_{t+1} = N_t + r_d N_t$$

$$N_{t+1} = N_t (1 + r_d)$$

let $1 + r_d = \lambda$, the finite rate of increase. Then:

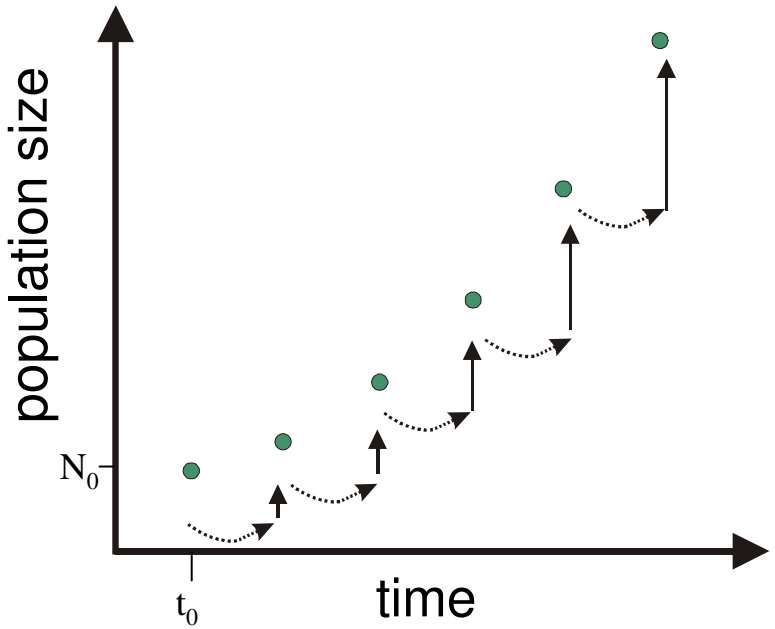
$N_1 = \lambda N_0$ → the « output » (N_{t+1}) forms the « input » (N_t)
$N_2 = \lambda N_1 = \lambda \lambda N_0 = \lambda^2 N_0$
...



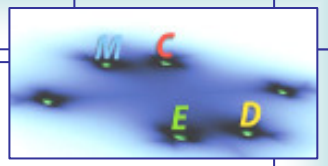
Difference equations (discrete systems)

→ the « output » (N_{t+1}) forms the « input » (N_t)

→ general solution to this recursion equation: $N_t = \lambda^t N_0$



(exponential growth)



Difference equations (discrete systems)

Examples

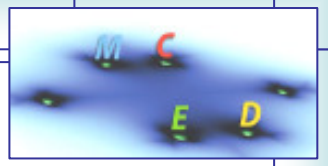
$N_{n+1} = N_n + 2.0$ $N_0 = 0$	$N_{n+1} = 2.0*N_n$ $N_0 = 1.0$	$N_{n+1} = 0.5*N_n^2$ $N_0 = 1.5$

Iteration tables

0	0.0
1	
2	
3	
4	
5	

0	1.0
1	
2	
3	
4	
5	

0	1.5
1	
2	
3	
4	
5	



Difference equations (discrete systems)

Examples

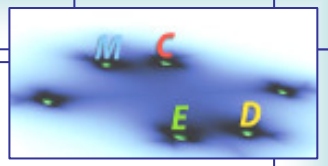
$N_{n+1} = N_n + 2.0$ $N_0 = 0$	$N_{n+1} = 2.0*N_n$ $N_0 = 1.0$	$N_{n+1} = 0.5*N_n^2$ $N_0 = 1.5$

Iteration tables

0	0.0
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0

0	1.0
1	2.0
2	4.0
3	8.0
4	16.0
5	32.0

0	1.5
1	1.125
2	0.633
3	0.2
4	0.02
5	0.002



Difference equations (discrete systems)

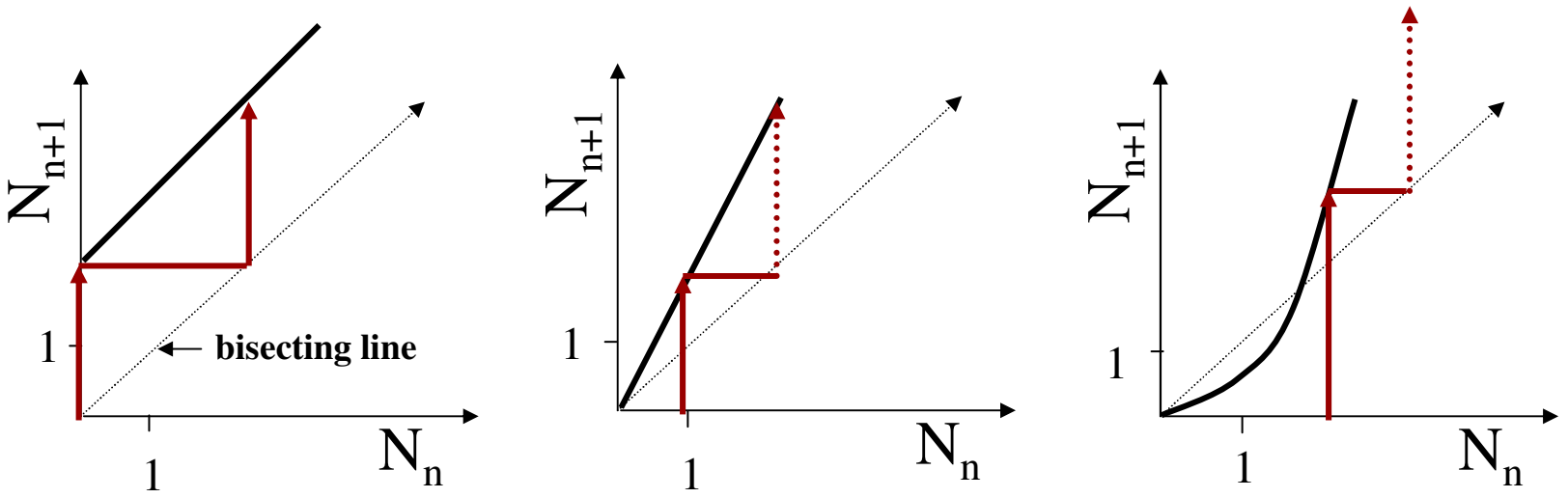
Examples

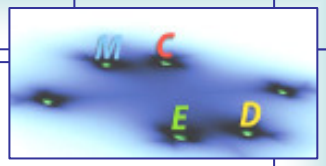
$N_{n+1} = N_n + 2.0$
$N_0 = 0$

$N_{n+1} = 2.0 * N_n$
$N_0 = 1.0$

$N_{n+1} = 0.5 * N_n^2$
$N_0 = 2.5$

Graphical iteration (« cobweb method »):

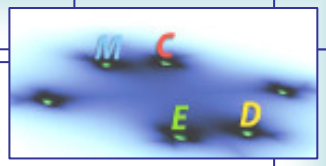




Difference equations (discrete systems)

Group exercises:

- ***Iterating difference equations***
- ***Plenary presentation and discussion***

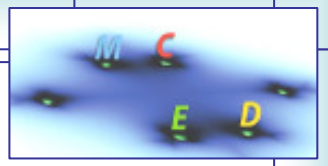


Tasks

Difference equations (discrete systems)

- *Group exercises: iterating difference equations*

Topics:	*Exponential growth* *Exponential decline* *Exponential growth with conditonal reduction*	*group 1*
	Logistic function	*group 2*
	Roof function	*group 3*

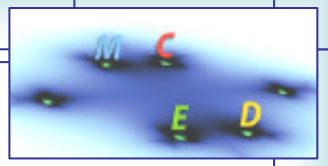


Tasks

Difference equations (discrete systems)

Task for all groups:

- ***write a table with two columns: iteration number, and obtained value. Calculate as far as reasonable***
- ***draw a graphic (15 x15 cm, 0.0 … 1.5)***
  and represent the values according to the cobweb method
- ***work on paper and after finishing transfer to an overhead transparency to demonstrate the results***



Tasks

Difference equations (discrete systems)

Group 1 ***Step-by-step calculation and graphical iteration***

Exponential growth

$Y_{(n+1)} = Y_n + 0.1\ Y_n$

$Y_{(initial)} = 0.1$

Exponential decline

$Y_{(n+1)} = Y_n - 0.1\ Y_n$

$Y_{(initial)} = 1.5$

Exponential growth with conditional reduction

if $Y_n <= 1.0$

then $Y_{(n+1)} = Y_n + 0.2\ Y_n$

else $Y_{(n+1)} = Y_n + 0.2\ Y_n - 0.5$ $Y_{(initial)} = 0.2$

Solution

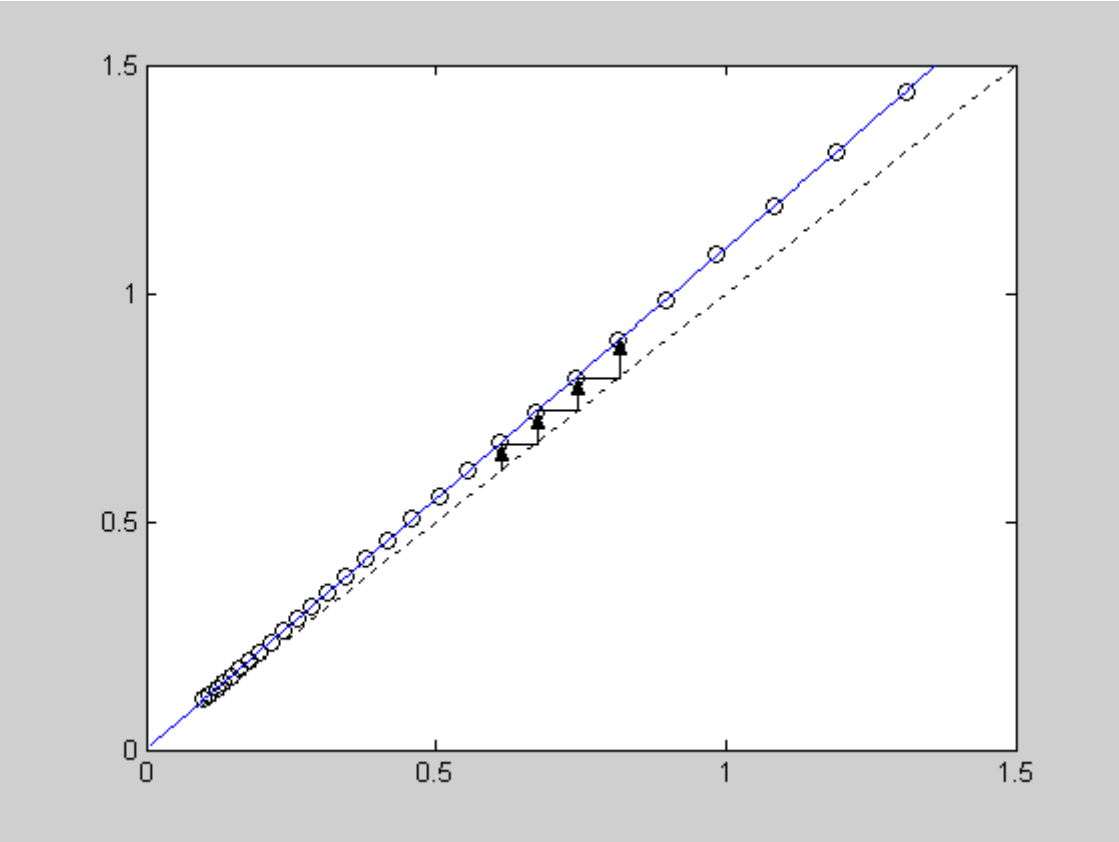
Difference equations (discrete systems)

Group 1 ***Step-by-step calculation and graphical iteration***

Exponential growth

$Y_{(n+1)} = Y_n + 0.1\ Y_n$

$Y_{(initial)} = 0.1$



Solution

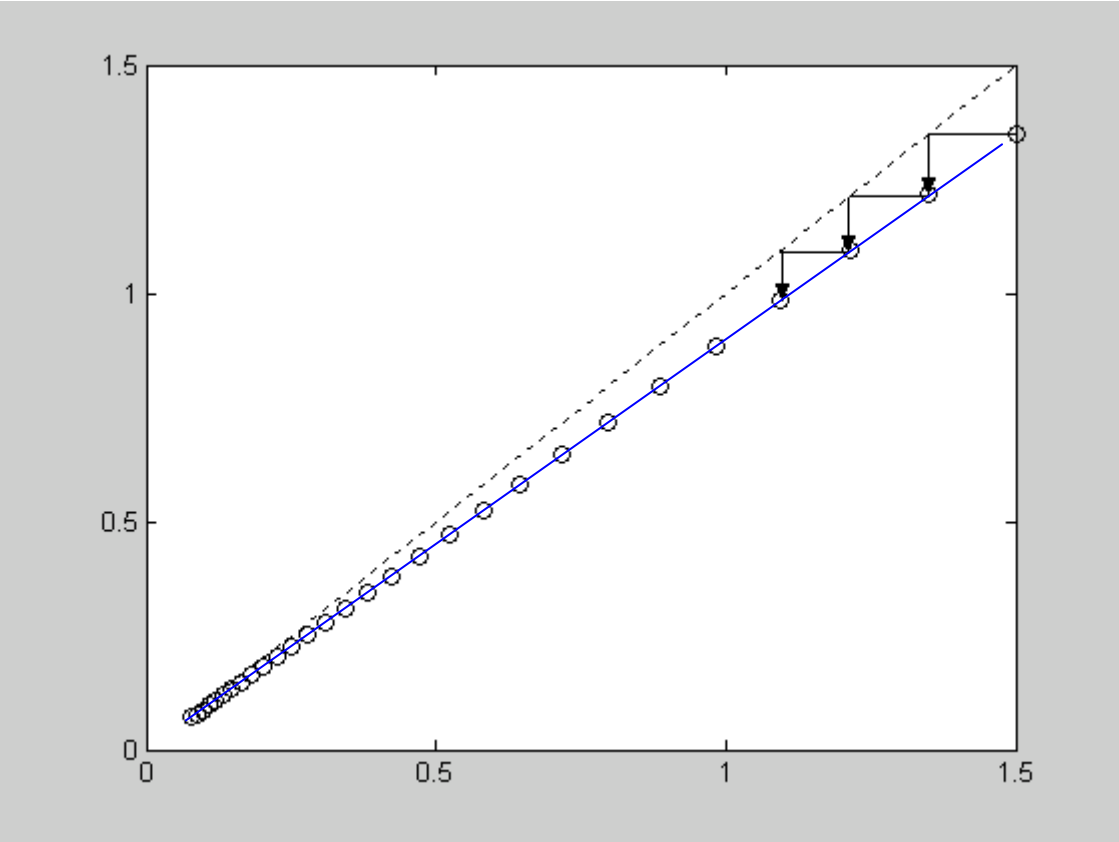
Difference equations (discrete systems)

Group 1 ***Step-by-step calculation and graphical iteration***

Exponential decline

$Y_{(n+1)} = Y_n - 0.1\ Y_n$

$Y_{(initial)} = 1.5$



Solution

Difference equations (discrete systems)

Group 1 ***Step-by-step calculation and graphical iteration***

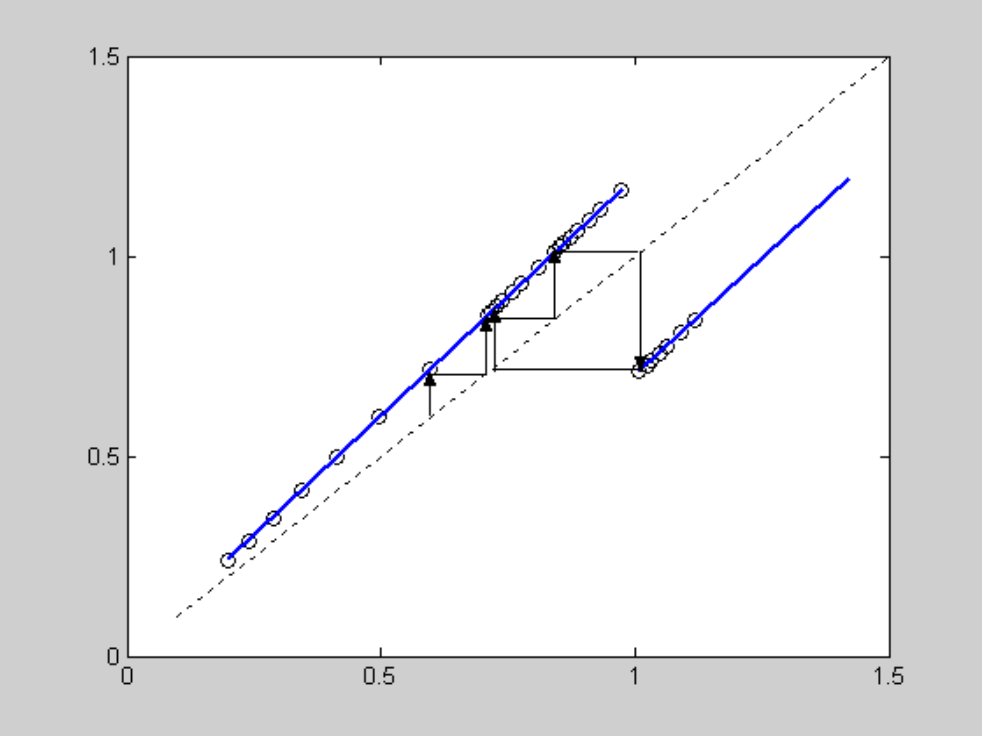
<u>Exponential growth with conditional reduction</u>

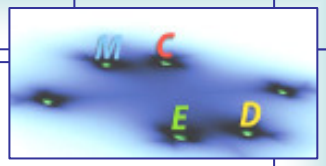
if $Y_n <= 1.0$

then $Y_{(n+1)} = Y_n + 0.2\ Y_n$

else $Y_{(n+1)} = Y_n + 0.2\ Y_n - 0.5$

$Y_{(initial)} = 0.2$





Task

Difference equations (discrete systems)

Group 2 ***Step-by-step calculation and graphical iteration***

Logistic Function

$$Y_{(n+1)} = Y_n + r*Y_n - r*Y_n^2$$
$$= Y_n + r*(Y_n - Y_n^2)$$

$$Y_{(initial)} = 0.4$$

Iterate function with r = 1.8
r = 2.4
r = 2.7

Solution

Difference equations (discrete systems)

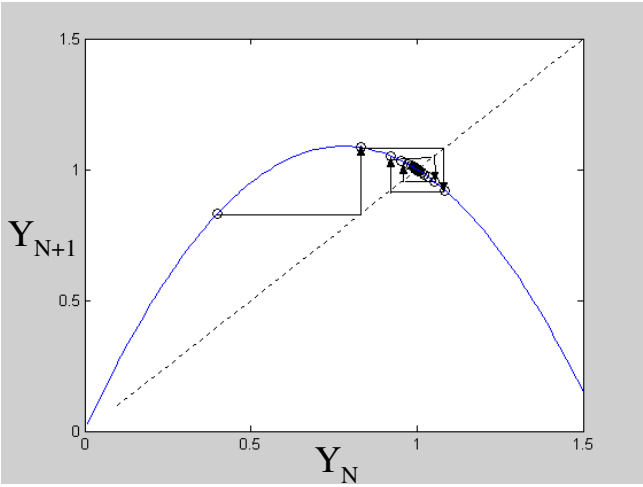
Group 2 ***Step-by-step calculation and graphical iteration***

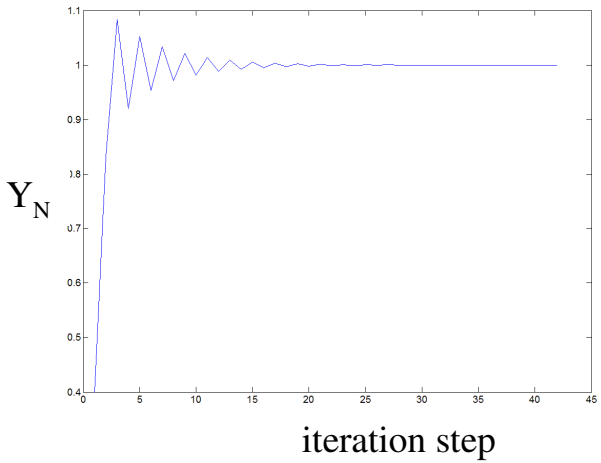
Logistic Function

$$Y_{(n+1)} = Y_n + r*Y_n - r*Y_n^2$$
$$= Y_n + r*(Y_n - Y_n^2)$$

$Y_{(initial)} = 0.4$

- ***r = 1.8***





Solution

Difference equations (discrete systems)

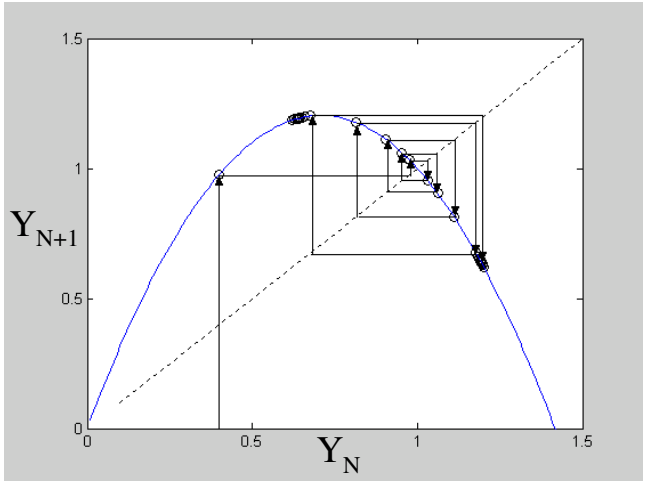
Group 2 ***Step-by-step calculation and graphical iteration***

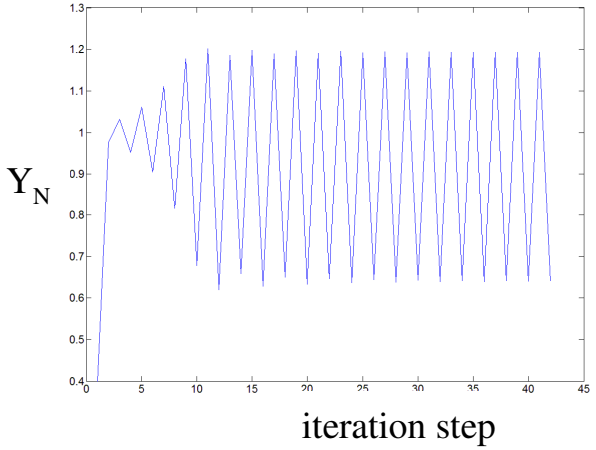
Logistic Function

$$Y_{(n+1)} = Y_n + r*Y_n - r*Y_n^2$$
$$= Y_n + r*(Y_n - Y_n^2)$$

$Y_{(initial)} = 0.4$

- ***r = 2.4***





Solution

Difference equations (discrete systems)

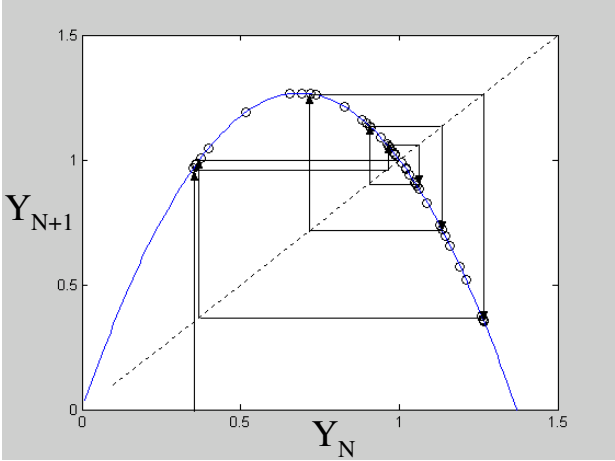
Group 2 ***Step-by-step calculation and graphical iteration***

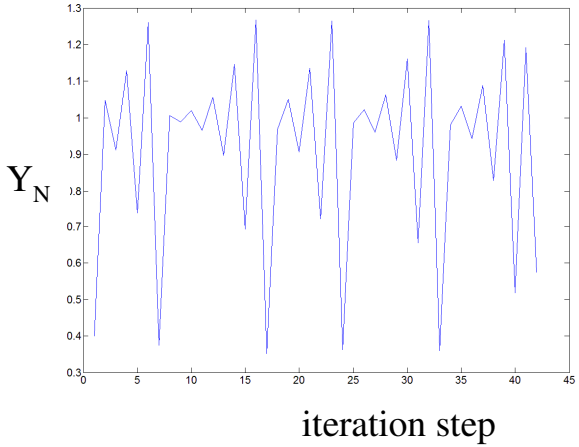
Logistic Function

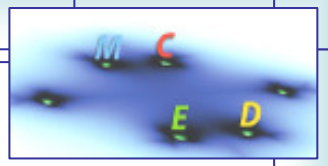
$$Y_{(n+1)} = Y_n + r*Y_n - r*Y_n^2$$
$$= Y_n + r*(Y_n - Y_n^2)$$

$$Y_{(initial)} = 0.4$$

- ***r = 2.7***







Task

Difference equations (discrete systems)

Group 3 ***Step-by-step calculation and graphical iteration***

Roof Function

if $Y_n <= 0.5$
then $Y_{(n+1)} = Y_n + r * Y_n$
else $Y_{(n+1)} = - Y_n * (1 + r) + (1 + r)$

Iterate function with

$Y_{(initial)} = 0.3$ $r = 0.1$
$Y_{(initial)} = 0.4$ $r = 0.5$
$Y_{(initial)} = 0.3$ $r = 0.9$

Solution

Difference equations (discrete systems)

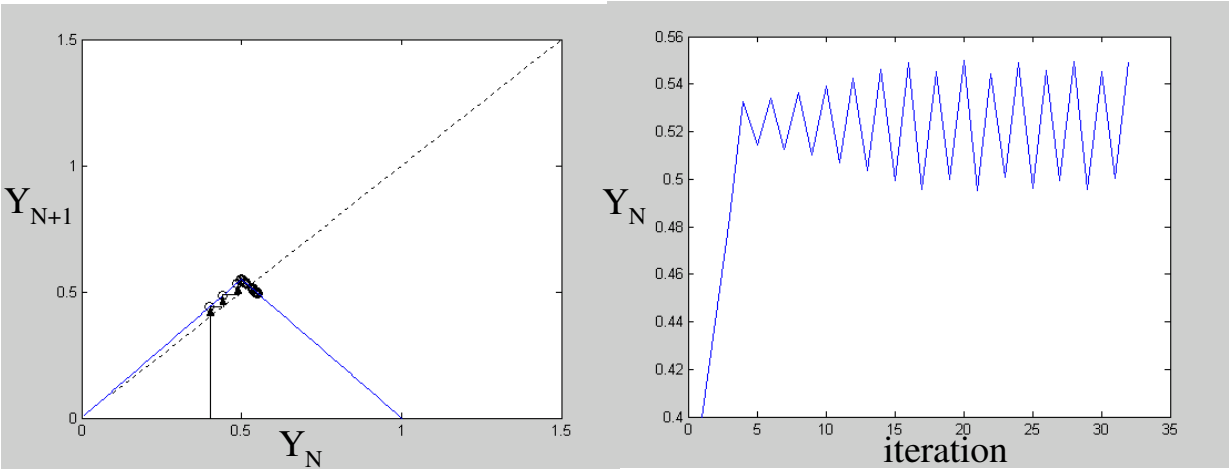
Group 3 ***Step-by-step calculation and graphical iteration***

Roof Function

if $Y_n <= 0.5$
then $Y_{(n+1)} = Y_n + r * Y_n$
else $Y_{(n+1)} = - Y_n * (1 + r) + (1 + r)$

$Y_{(initial)} = 0.3$

- ***r = 0.1***



Solution

Difference equations (discrete systems)

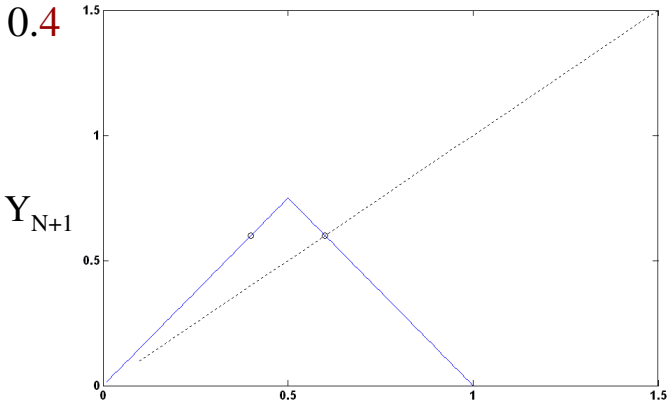
Group 3 ***Step-by-step calculation and graphical iteration***

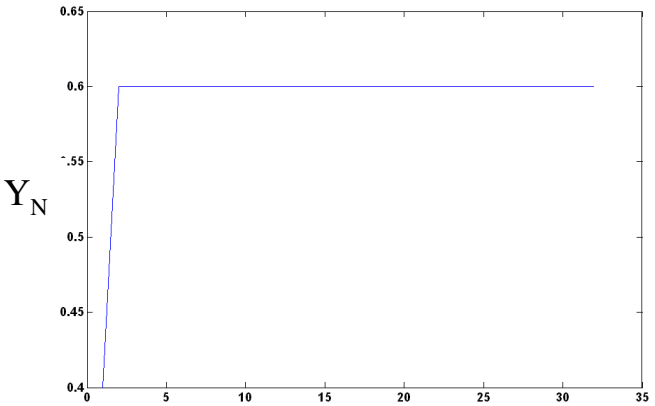
Roof Function

if $Y_n <= 0.5$
then $Y_{(n+1)} = Y_n + r * Y_n$
else $Y_{(n+1)} = - Y_n * (1 + r) + (1 + r)$

$Y_{(initial)} = 0.4$

- ***r = 0.5***





Solution

Difference equations (discrete systems)

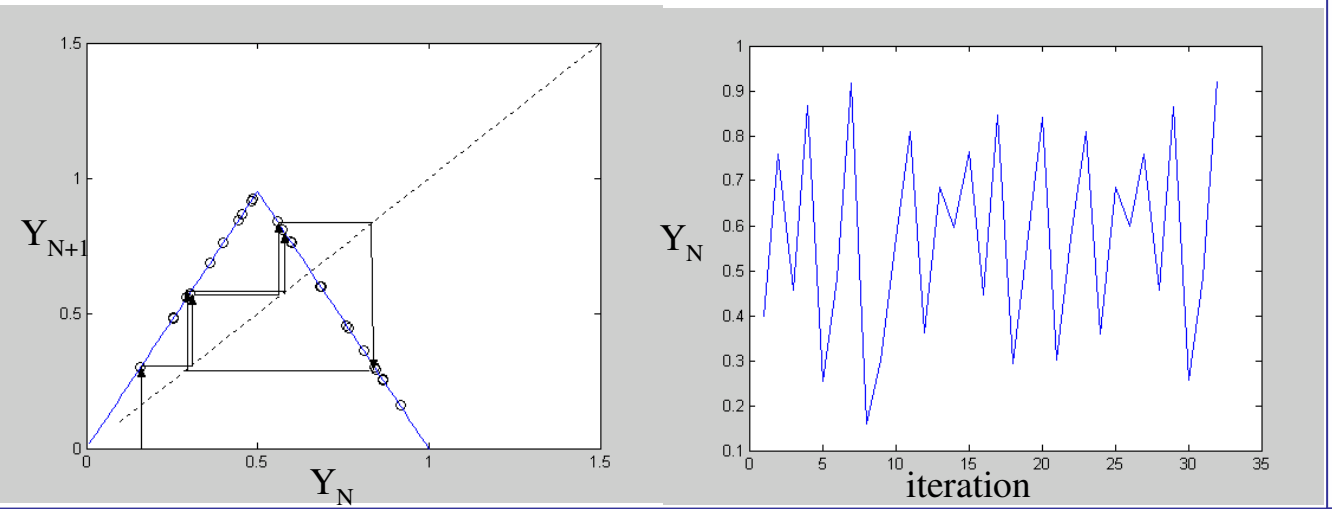
Group 3 ***Step-by-step calculation and graphical iteration***

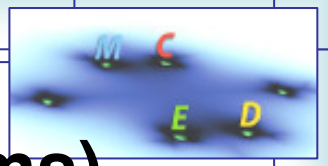
Roof Function

if $Y_n <= 0.5$
then $Y_{(n+1)} = Y_n + r * Y_n$
else $Y_{(n+1)} = - Y_n * (1 + r) + (1 + r)$

$Y_{(initial)} = 0.3$

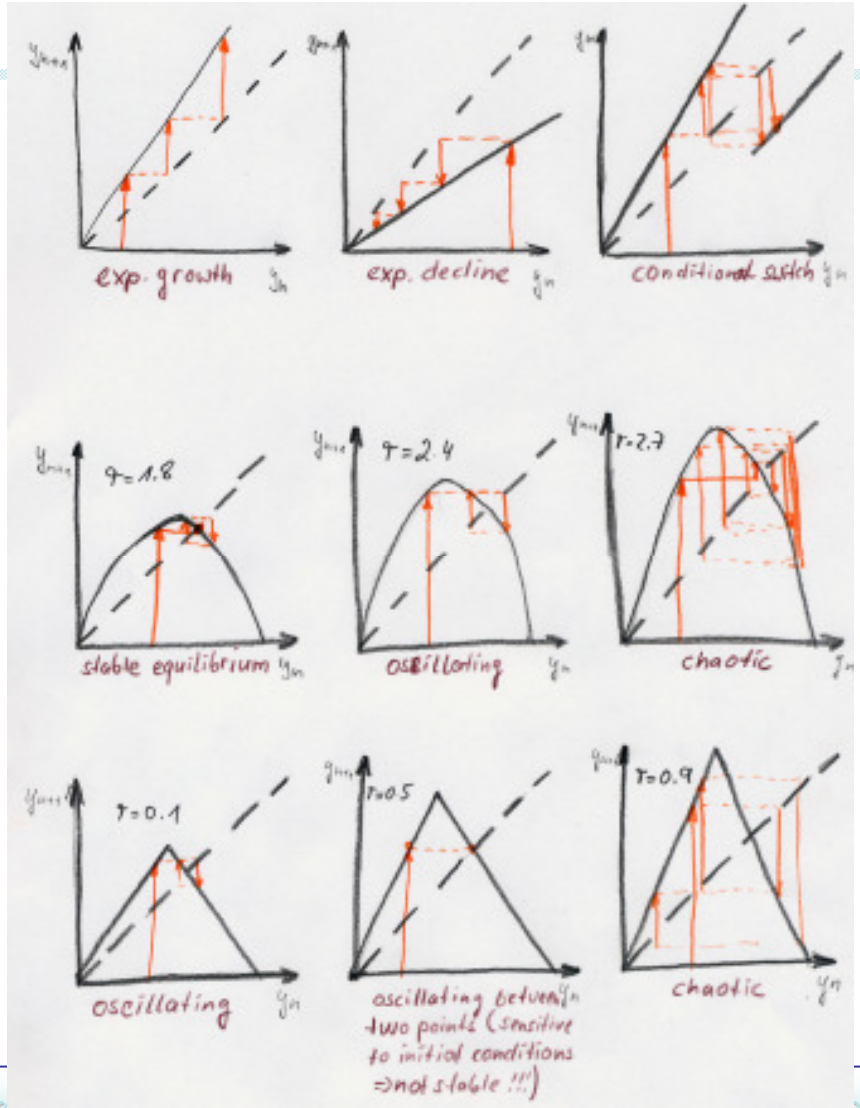
- ***r = 0.9***

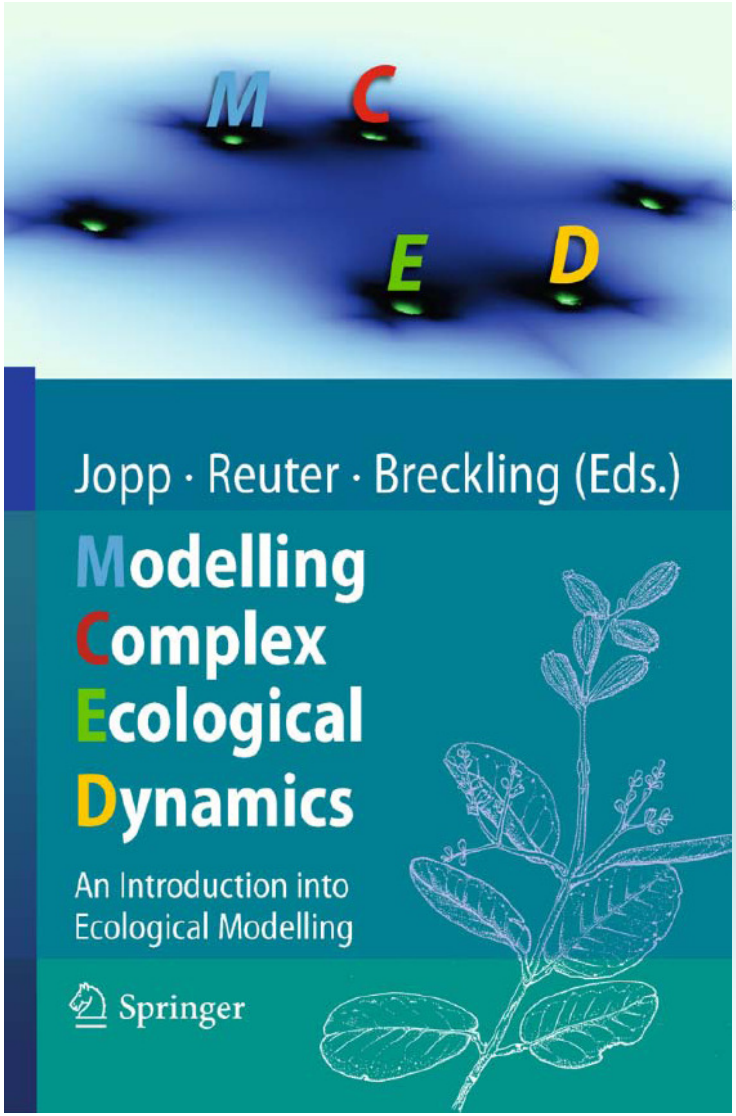




Difference equations (discrete systems)

survey sketch





This is it
for now,

but the fun
continues

…